Adapting to Contacts: Energy Tanks and Task Energy for Passivity-Based Dynamic Movement Primitives

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Abstract—In this paper, we develop a framework to encode demonstrated trajectories as periodic dynamic motion primitives (DMP) for an impedance-controlled robot and their modification to fulfill the task objective, i.e., to adapt based on the force feedback and encoded desired wrench profile via an admittance controller. This behavior by itself can violate stability. Therefore, a passivity analysis for the whole system is presented, and based on input power ports and the demonstrated reference power, a passivity observer (PO) is designed. Subsequently, a DMP phase altering law is introduced according to the passivity criterion in order to adjust the phase based on the passivity criterion. However, since this does not necessarily guarantee passivity, a suitable virtual energy tank is used. Experimental results on a Kuka LWR-4 robot polishing an unknown surface underline the real-world applicability of the suggested controller.

I. INTRODUCTION

Transfer of knowledge from a human to a robot has been thoroughly investigated in robotics [1]. However, transfer of motion to achieve a task is only meaningful with matching correspondences – if we demonstrate squatting a robot, it should perform squatting, not move legs while falling over [2]. Transfer of knowledge has been applied to many different tasks humans perform in various environments, including households. Wiping of surfaces, e.g., is one of the most common household tasks, showing semantic similarity with numerous actions [3]. Consequently, it is no surprise that wiping – rubbing on a surface with a cloth or sponge – has been extensively explored in robotics literature. As stated in [4], wiping demands rich and complex actions, which can result in different effects depending on the concrete execution - it may be plain wiping, or even polishing, painting or removing of material (grinding). Our experimental humanoid robotic polishing setup is depicted in Fig. 1.

Just as many actions can be semantically similar to wiping, many different approaches have been described for wiping in the robotics literature, ranging from kinematic [5] and dynamic approaches [6], to high-level of abstraction [4], [7] and dealing with deformable objects [8]. Within learning by demonstration (LbD), methods combining learning and force control within the scope of adaptive LbD have been used to acquire and maintain contact on arbitrary surfaces [9].

While many papers deal with the execution of wiping tasks as the appropriate use-case scenario, only few deal with energy transfer between robot and environment in the scope of maintaining system stability and optimality for both robot and environment, respectively. Ensuring stability is tightly related with the concept of passivity, which essentially refers to a system’s property not to produce more energy than it receives. A useful feature of the concept is its modularity [10], which makes it possible to divide a system into subsystems with properly chosen interconnections and after analyzing the passivity of each subsystem, conclude overall system passivity.

In this paper, we embed adaptive LbD methods into the concept of passivity-based control in order to design and prove overall system stability during complex tasks such as tactile wiping. The considered problem statement and related work are introduced next, leading to the overall contributions made.

A. Problem Statement

Let us consider a system that is able to execute motions demonstrated by a human operator or that were generated by associated planning algorithms, while adapting these based on force feedback to maintain desired contact behavior with environment. Our overall goal is that the system is able to

- adapt its speed at high performance based on an observer that monitors system passivity in real-time.
- ensure overall system stability based on the continuous transfer of energy between the involved subsystems.

B. Related work

Related work can be separated into two main topics: 1) stability in learning by imitation and 2) passivity-based control.

Learning by imitation has been tackled in different manners, one of which is with the use of dynamic movement primitives (DMPs) [11], [12]. DMPs per se have been shown stable [11], however, this does not imply the stability of in-contact robotic operations. For example, stability of learning
for maintaining environmental contact through DMP coupling terms has been separately investigated in [13]. DMPs are of course not the only means of encoding trajectories. For example, Gaussian Mixture Models (GMMs) can be used [14], [15]. GMMs were used to estimate the entire attractor landscape of a movement skill from several demonstrations. While the authors showed how to maintain the stability of the dynamical system, they did not specifically tackle the stability of in-contact behavior of the system. More recently, variable stiffness was applied as one of possible approaches in combination with motion imitation to ensure safety of interaction [16].

In our current work we show how the overall system stability of in-contact robot behaviors can be maintained through observing the passivity of the system.

Passivity-based control in humanoid robots was mostly considered for gait stabilization and posture control [17]–[19]. For impedance controlled robots passivity was investigated for contact free scenarios [20] and the case of regulation [21]. When considering following desired force profiles with impedance controlled robots, the recent approach of unified force/impedance control proposed in [22] allows for accurate force tracking and simultaneous impedance control. In the current paper, an admittance controller is used to adapt the previously learned trajectory to respect the desired force profile. Several works have proposed the combination of admittance and motion control [23]–[25], among those most are for teleoperation applications and followed different goals. Moreover, although some have applied passivity analysis and specifically the concept of reference energy [26], [27], none has modeled the overall system from an energy and passivity point of view such that the encoded power can be interpreted as a system with its own storage function. Finally, the augmentation of virtual energy tanks has drawn significant attention for stability analysis in robotics [22], [28], [29], and is used in this paper for passivity proof as well as robustness in task performance.

C. Contributions

The contributions of the present work are as follows.

1) Passivity analysis for desired trajectory and wrench encoding based on the concept of demonstrated reference power.
2) Design and validation of a passivity observer (PO) for online phase altering of encoded trajectory and wrench profile.
3) Design and validation of a passivity enforcing virtual energy tank for task robustness.

The remainder of the paper is organized as follows. In Section II the basic models and notations for Cartesinan Impedance controlled robots with DMP trajectory generation are reviewed. Section III explains the trajectory modification scheme based on admittance control. Passivity analysis of the proposed control subsystems follows in Section IV where it is shown that trajectory adaptation based on force feedback via admittance control is not always passive. Section V explains how system passivity can be observed and accordingly reacted to via reference velocity adaptation in case of violation. Finally, a virtual tank is introduced to guarantee the overall passivity. Experimental evaluation of the proposed passive interaction based on a surface polishing task is presented in Section VI, with conclusions given in Section VII.

II. PRELIMINARIES

In this section, a Cartesian Impedance controlled robot for which the desired trajectory is encoded via the well known DMP formulation is modeled.

A. Rigid-body Robot Model

Considering the robot pose \( \mathbf{x} = (\mathbf{p}^T, \varphi^T)^T \in \mathbb{R}^6 \) where \( \mathbf{p} \in \mathbb{R}^3 \) is the translational part and \( \varphi \in \mathbb{R}^3 \) is the Euler angles representing the rotational part, dynamics of rigid body robots in Cartesian space with \( n \) degrees of freedom may be written as

\[
M_K(q)\ddot{\mathbf{x}} + C_K(q, \dot{q})\dot{\mathbf{x}} + F_g(q) = F_m + F_{\text{ext}},
\]

(1)

where \( q \in \mathbb{R}^n \) denotes the link position and \( \dot{q} \in \mathbb{R}^n \) the link velocity. \( F_{\text{ext}} \in \mathbb{R}^6 \) is the external wrench applied to the robot end-effector and obtained from wrist sensing. It consists of translational forces \( f_{\text{ext}} \in \mathbb{R}^3 \) and rotational moments \( m_{\text{ext}} \in \mathbb{R}^3 \). The controller input \( \tau_m \in \mathbb{R}^n \) and the robot gravitational vector \( g(q) \in \mathbb{R}^n \) relate to the Cartesian space wrenches \( F_m, F_g(q) \in \mathbb{R}^6 \) by

\[
\tau_m = J^T(q)F_m
\]

(2)

\[
g(q) = J^T(q)F_g(q),
\]

(3)

where \( J(q) \) is the robot Jacobian matrix. \( M_K(q) \in \mathbb{R}^{n \times n} \) and \( C_K(q, \dot{q}) \in \mathbb{R}^{n \times n} \) are the inertia matrix and the Coriolis and centrifugal matrix of the robot in Cartesian space, which are related to the equivalent joint quantities \( M(q) \in \mathbb{R}^{n \times n} \) and \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) via [31]

\[
M_K(q) = J^T(q)M(q)J^T(q)
\]

(4)

\[
C_K(q, \dot{q}) = \left( J^T(q)C(q, \dot{q}) - M_K(q)J(q) \right) J^T(q).
\]

(5)

Here, the right-hand Moore-Penrose pseudo inverse \( J^\dagger(q) = J^T(q)(J(q)J^T(q))^{-1} \) is used for inverting the Jacobian matrix.

B. Cartesian Impedance control

In order to have a closed loop Cartesian impedance control with the desired stiffness \( K_x \in \mathbb{R}^{6 \times 6} \), damping \( D_x \in \mathbb{R}^{6 \times 6} \) and inertia identical to the robot inertia \( M_K(q) \), the impedance control law becomes [30]:

\[
\tau_m = -J^T(q) \left[ K_x \ddot{\mathbf{x}} + D_x \dot{\mathbf{x}} - M_K(q)\dot{x}_d - C_K(q, \dot{q}) \dot{x}_d \right] + g(q),
\]

(6)

where \( \dot{x}_d := \dot{x} - \dot{x}_d \) and \( x_d \in \mathbb{R}^6 \) is the desired robot pose. As a result, considering (1) and (6), the closed loop dynamics becomes

\[
M_K(q)\ddot{\mathbf{x}} + (C_K(q, \dot{q}) + D_x)\dot{\mathbf{x}} + K_x \ddot{\mathbf{x}} = F_{\text{ext}}.
\]

(7)

At this point it shall be mentioned that due to the modularity of the passivity concept one may choose any other controller that together with the rigid-body robot results in a passive system w.r.t. the relative ports explained in Sec. IV-A.

1Please note that in this work, only rigid-body robots are considered. For modeling flexible-joint robots see e.g. [30].

2If the left-hand pseudo inverse of \( J^\dagger(q) = (J^T(q)J(q))^{-1}J^T(q) \) is used, \( J^\dagger(q) \) should be replaced by \( J^T(q) \).
C. Dynamic Movement Primitives

Dynamic movement primitives are an elegant approach of encoding trajectories, typically used in learning by demonstration. In the following we provide a brief recap of periodic DMPs, which are used in our control scheme. Further details about DMPs can be found [11].

A periodic DMP, which is in our case one of the task-space variables as introduced in Section II-A, is given by

\[
\dot{x}_{\text{DMP}} = \Omega \hat{\phi},
\]

(9)

where \( \Omega \) is the frequency of oscillations and \( f(\phi) \) denotes the periodic forcing term.

Encoding of the periodic forcing term uses a linear combination of \( N \) periodic basis functions \( \Gamma_i \)

\[
f(\phi) = \sum_{i=1}^{N} u_i \Gamma_i(\phi) \sum_{i=1}^{N} \Gamma_i(\phi) r_i,
\]

(11)

\[
\Gamma_i(\phi) = \exp(h_i(\phi - c_i - 1)),
\]

(12)

where \( r \) is the amplitude of the oscillator and \( h_i > 0 \). Throughout this work, we use \( N = 25 \) and \( h_i = 2.5 N \). The movement frequency \( \Omega \) can be automatically determined from the data [12], [33].

III. Trajectory Modification via Admittance Control

While the Cartesian impedance controller (6) is responsible for empowering the closed loop behavior (7), the higher level goal would be to adapt the trajectory reference \( \dot{x}_d \) with respect to environmental contacts. A common example is to respect a desired end-effector wrench profile \( F_d \in \mathbb{R}^6 \) while performing a desired motion. Obviously, if the desired pose trajectory is encoded as a DMP, the desired wrench representation should also share the same phase function as the DMP. Here, we propose that the according modification may be done via the following standard admittance controller.

\[
F_d + F_{\text{ext}} = M_\alpha \ddot{x}_a + D_\alpha \dot{x}_a + K_\alpha x_a
\]

(13)

Its dynamics are governed by a mass-spring-damper behavior with \( M_\alpha, D_\alpha, K_\alpha \) being the desired inertia, damping and stiffness, respectively. Overall, the system has the input \( F_d + F_{\text{ext}} \) and output \( x_a \in \mathbb{R}^6 \), which encodes the DMP adaptation.

\[
x_d = x_{\text{DMP}} + x_a
\]

(14)

Figure 2 depicts the overall control architecture. It should be mentioned that while any other trajectory generator could be used, here DMP’s are exploited due to their convenience in reference motion encoding on the one hand, and on the other hand the intuitive way of reference velocity adaptation based on the passivity criterion introduced in section V.

Fig. 2: Block diagram of the overall system, consisting of DMP and desired wrench which share the same phase function, the admittance controller which modifies the reference trajectory, the Cartesian impedance controller, the rigid body robot and the environment. Moreover the effect of the energy tank on the system is depicted as dotted lines.

IV. Preliminary Passivity Analysis

In order to monitor stability of the overall system while interacting with environment, we rely on the concept of passivity-based control. As noted above one of the important properties of passivity analysis is that the proper interconnection of passive subsystems leads to an overall passive system. After the following definition of passivity each subsystem is analyzed and the overall system passivity is investigated in the end.

**Definition:** A system with the state space model of \( \dot{\chi} = f(\chi, u) \) with initial state of \( \chi(0) = \chi_0 \in \mathbb{R}^m \), input vector \( u \in \mathbb{R}^l \) and output \( y = h(\chi, u) \) is said to be passive, if there exists a positive semidefinite function \( S : \mathbb{R}^m \rightarrow \mathbb{R}_+ \), called storage function, such that

\[
S(\chi(\sigma)) - S(\chi_0) \leq \int_0^\sigma u^T(t)y(t)dt
\]

(15)

for all input signals \( u : [0, \sigma] \rightarrow \mathbb{R}^l \), initial states \( \chi_0 \in \mathbb{R}^m \) and \( \sigma > 0 \). Thus proving passivity of a system is equivalent to finding an appropriate storage function \( S(\chi) \) such that

\[
\dot{S} \leq u^T y \quad \forall (\chi, u).
\]

(16)

A. Subsystem passivity

a) Environment: In passivity-based analysis there is no need for an exact system model, since it is commonly assumed that the environment is passive w.r.t. the pair \( \langle \dot{x}, -F_{\text{ext}} \rangle \). In other words, there exists a storage function \( S_{\text{env}} \) such that

\[
\dot{S}_{\text{env}} \leq -\dot{x}^T F_{\text{ext}}.
\]

(17)

b) Cartesian impedance controlled robot: Considering (7), the proposed storage function for the super-system of Cartesian impedance controller and rigid body robot is

\[
S_m = \frac{1}{2} \dot{x}^T K_\alpha \dot{x} + \frac{1}{2} \dot{x}^T M_\alpha(q) \dot{x} + \frac{1}{2} \dot{x}^T M_K(q) \dot{x}
\]

(18)

Its time derivative is

\[
\dot{S}_m = \dot{x}^T K_\alpha \dot{x} + \frac{1}{2} \dot{x}^T M_\alpha(q) \dot{x} + \dot{x}^T M_K(q) \dot{x}
\]

\[
= \dot{x}^T K_\alpha \dot{x} + \frac{1}{2} \dot{x}^T M_\alpha(q) \dot{x}
\]

\[
+ \dot{x}^T (-C_K(q, \dot{q}) \dot{x} - D_\alpha \dot{x} - K_\alpha \dot{x} + F_{\text{ext}})
\]

\[
= \dot{x}^T F_{\text{ext}} - \dot{x}^T D_\alpha \dot{x} + \frac{1}{2} \dot{x}^T \left( M_\alpha(q) + 2C_K(q, \dot{q}) \right) \dot{x}
\]

\[
\leq 0 \implies \dot{S}_m \leq \dot{x}^T F_{\text{ext}} = \dot{x}^T F_{\text{ext}} - \dot{x}^T_{\text{d}} F_{\text{ext}},
\]

(19)
where the skew-symmetry of the matrix $\dot{M}_K(q) - 2C_K(q, \dot{q})$ is taken into account. As a result, the Cartesian impedance controlled robot is passive w.r.t. the pair $(\dot{x} - \dot{x}_d, F_{\text{ext}})$. In case of variable stiffness (19) cannot be achieved, and the time-derivative of $K_x$ has to be considered [34].

c) Admittance control: The storage function for the admittance controller (13) may be defined as

$$S_a = \frac{1}{2} \dot{x}_a^T M_a \dot{x}_a + \frac{1}{2} x_a^T K_a x_a. \quad (20)$$

The time derivative of this function is

$$\dot{S}_a = \dot{x}_a^T (M_a \dot{x}_a + K_a x_a) = \dot{x}_a^T (F_d + F_{\text{ext}}) - \dot{x}_a^T D_a \ddot{a}_a \geq 0,$$

$$\implies \dot{S}_a \leq \dot{x}_a^T F_d + \dot{x}_a^T F_{\text{ext}}. \quad (21)$$

Consequently, the admittance control law system is passive w.r.t. the pair $(\dot{x}_a, (F_d + F_{\text{ext}})).$

B. Passivity of overall system

By defining the overall storage function as

$$S_{\text{tot}} = S_{\text{env}} + S_m + S_a, \quad (22)$$

passivity of the overall system as an integration of the aforementioned subsystems can be investigated by analyzing the time derivative of $S_{\text{tot}}$. Considering (17), (19) and (21), and replacing $x_d$ with its equivalence via (14) it can be concluded that

$$\dot{S}_{\text{tot}} = \dot{S}_{\text{env}} + \dot{S}_m + \dot{S}_a$$

$$\leq \dot{x}_a^T F_{\text{ext}} - \dot{x}_a^T F_{\text{DMP}} + \dot{x}_a^T F_{\text{ext}} + \dot{x}_a^T F_{\text{DMP}}$$

$$\leq \dot{x}_a^T F_d + \dot{x}_a^T F_{\text{DMP}}. \quad (23)$$

Hence, the overall system is passive w.r.t. the pairs $(\dot{x}_a, F_d)$ and $(\dot{x}_{\text{DMP}}, -F_{\text{ext}})$, see Fig. 3.

The deduced passivity does not necessarily imply the stability of the system, as one should also pay attention to the power being injected to the system through the ports $(\dot{x}_a, F_d)$ and $(\dot{x}_{\text{DMP}}, -F_{\text{ext}})$. We propose that this power should be compared with a reference power trajectory that is computed from the desired velocity and force commands as shown next.

C. Reference energy

Considering the desired encoded motion $x_{\text{DMP}}^\dagger$ with its desired task frequency $\Omega^\dagger$ and the desired wrench profile $F_d^\dagger$, both being defined by the user, the desired power trajectory becomes

$$P_{\text{in}}^\dagger := \dot{x}_{\text{DMP}}^\dagger F_d^\dagger. \quad (24)$$

$P_{\text{in}}^\dagger$ can be considered as power coming from a passive system with the storage function $S^\dagger$ such that according to (16)

$$\dot{S}^\dagger \leq -P_{\text{in}}^\dagger. \quad (25)$$

Considering (23) and (25), it is straight-forward to see that

$$S_{\text{tot}} + \dot{S}^\dagger \leq \dot{x}_a^T F_d - \dot{x}_{\text{DMP}} F_{\text{ext}} - P_{\text{in}}^\dagger. \quad (26)$$

Thus, for the overall passivity it is enough to show that

$$\dot{x}_a^T F_d - \dot{x}_{\text{DMP}}^T F_{\text{ext}} - P_{\text{in}}^\dagger \leq 0. \quad (27)$$

$F_d$ is assigned in the beginning, $F_{\text{ext}}$ and consequently $\dot{x}_a$ are not modifiable. $P_{\text{in}}^\dagger$ is always known after defining the desired motion via (24). Therefore, the only possibility to keep (27) valid is to modify the norm of $\dot{x}_{\text{DMP}}$ through its phase function by changing the task period.

In the next section, we introduce a new scheme to determine the phase function of $\dot{x}_{\text{DMP}}$ such that the system intends to improve performance subject to remaining passive.

V. SYSTEM PASSIFICATION

A. Passivity observer (PO)

Based on (27), a passivity criterion can be found, by which system passivity can be observed at all times. For this, the power $P_{\text{acv}}$ is defined as

$$P_{\text{acv}}(t) = \dot{x}_a^T(t) F_d(\phi) - \dot{x}_{\text{DMP}}^T F_{\text{ext}}(t) - P_{\text{in}}^\dagger(\phi). \quad (28)$$

where $\phi$ is obtained via (10). Based on the sign of $P_{\text{acv}}(t)$ the proposed passivity observer becomes

$$\begin{cases} \text{Passive} & P_{\text{acv}}(t) \leq 0 \\ \text{Non-passive} & P_{\text{acv}}(t) > 0 \end{cases} \quad (29)$$

Note that ideally one can assume $F_{\text{ext}} = -F_d$, leading to $\dot{x}_a = 0$. Consequently, $P_{\text{acv}}(t) = 0$ holds ideally.

B. DMP phase-function assignment

The output velocity of the DMP can be changed according to (29) such that if passivity is being violated, $\dot{x}_{\text{DMP}}$ is reduced to let the system tend back to show passive behavior. Moreover, if the system is already passive with some margin, $\dot{x}_{\text{DMP}}$ may be increased. Such performance may be realized by changing the phase function (10) through assigning a task frequency $\Omega$ via

$$\Omega = \Omega^\dagger - KP_{\text{acv}}, \quad (30)$$

where $\Omega^\dagger$ denotes the original task frequency, $\Omega$ is the primary assigned task frequency and $K$ is a positive-definite scalar quantity. As shown in Sec. VI, using this approach will bring $P_{\text{acv}}$ towards zero. However, this policy is not necessarily sufficient to ensure overall system passivity. Note that instead of DMP other trajectory generators could be used if their velocity can be accessed similarly.

\[4\] In this paper we do not consider variable-gain admittance control.

\[5\] as long as $F_{\text{ext}}$ is not in the null-space of $\dot{x}_{\text{DMP}}$. 

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Fig. 3: Port-based modelling of the subsystems and their relative power variables.
Fig. 4: The super-system with storage function $S_\Sigma$ consisting of energy tank, reference energy and overall system of admittance control + impedance controlled robot + environment.

C. Energy tank augmentation

Changing DMP output velocity via (30) does not necessarily ensure passivity of system. In order to guarantee passivity a virtual energy tank is introduced that takes care of potential passivity violating powers in (28). The tank is designed such that it provides the system with potentially demanded extra energy (i.e. $P_{acv}(t) > 0$) up to a limited amount of energy. This way, not only passivity is preserved, but there exists an extra robustness margin for system in case a reasonable amount of additional energy is required to perform the task. Finally, if this energy is not enough, the velocity decreases until system stop.

The tank energy $E_t$ associated to its system state $x_t$ is defined as

$$E_t = \frac{1}{2} x_t^2.$$  \hfill (31)

The tank dynamics are defined as

$$\dot{x}_t = \frac{\alpha_t}{x_t}(\gamma_t - 1)P_{acv} - \frac{\beta_t}{x_t}\gamma_t P_{acv},$$ \hfill (32)

where $\gamma_t$ is defined as

$$\gamma_t = \begin{cases} 1 & \text{if } P_{acv} \leq 0 \\ 0 & \text{else}. \end{cases}$$ \hfill (33)

$\beta_t$ is responsible to keep the energy tank below the upper limit $E_{up}$ and is defined as

$$\beta_t = \begin{cases} 1 & \text{if } E_t < E_{up} \\ 0 & \text{else}. \end{cases}$$ \hfill (34)

$\alpha_t$ keeps the energy tank above the lower limit $E_{low}$ and is defined as

$$\alpha_t = \begin{cases} 1 & \text{if } E_{low} + \delta_E \leq E_t \\ \frac{1}{2} \left[1 - \cos \left( \frac{E_t - E_{low}}{\delta_E} \pi \right) \right] & \text{if } E_{low} \leq E_t < E_{low} + \delta_E \\ 0 & \text{else}, \end{cases}$$ \hfill (35)

where $\delta_E$ is the threshold above the lower limit, in which the possible energy that can be taken from the tank reduces to zero when $E_t = E_{low}$.

By setting the initial tank energy $E_{t0} = E_t(t = 0)$ the tank scales the task frequency as follows.

$$\bar{\Omega} = \alpha_t \Omega$$ \hfill (36)

$\bar{\Omega}$ is the final DMP task frequency. Moreover, as dynamics of the tank also depend on the output velocity of the admittance control $\dot{x}_a$, the tank energy level will also have an effect on the admittance controller via

$$\dot{x}_a = \alpha_t \dot{x}_a,$$ \hfill (37)

where $x_a$ is the admittance control output obtained from (13), and $\dot{x}_a$ is the final value before sending to the system via (14). Hence, if there is not enough energy in the tank, $\alpha_t$ will decrease to zero, and consequently the output velocity of the DMP and admittance controller will decay to zero, eventually stopping the whole system.

Considering (32) to (37), as long as $\beta_t \neq 0$, it is straightforward to see that

$$\dot{E}_t = -P_{acv}.$$ \hfill (38)

Thus, a super-system consisting of the overall system storage function $S_{tot}$, the reference energy $S^r$ and tank energy $E_t$ can eventually be defined, see Fig. 4:

$$S_\Sigma = S_{tot} + S^r + E_t$$ \hfill (39)

Considering (23), (25), (28) and (38) one gets

$$\dot{S}_\Sigma = \dot{S}_{tot} + \dot{S}^r + \dot{E}_t \leq \dot{x}_a^T F_d - \dot{x}_D^T F_{ext} - P_{in}^T - P_{acv} = 0.$$ \hfill (40)

Moreover, if $\beta_t = 0$ the tank power $\dot{E}_t$ becomes

$$\dot{E}_t = \begin{cases} 0 & \text{if } P_{acv} \leq 0 \\ -P_{acv} & \text{else}. \end{cases}$$ \hfill (41)

and (40) turns into

$$\begin{cases} \dot{S}_\Sigma \leq P_{acv} & \text{if } P_{acv} \leq 0 \\ \dot{S}_\Sigma \leq 0 & \text{else}. \end{cases} \Rightarrow \dot{S}_\Sigma \leq 0$$ \hfill (42)

Thus, the super-system is passive. The phase modification approach and the tank effect are depicted in Fig. 5.

VI. EXPERIMENTAL RESULTS

The proposed method is tested on a 7 DOF Kuka LWR-4 impedance controlled robot, see Fig. 8. The considered challenge is to polish a surface, which location may alter during process execution. External wrenches $F_{ext}$ are measured with an additional ATI force/torque sensor mounted on the wrist of the robot arm. In this experiment the task is to maintain the normal contact force $F_d = 10$ N with
the table. Therefore, only forces in global z-direction are relevant. The upper and lower energy limit of the tank were chosen to be $E_{\text{up}} = 0.6$ J and $E_{\text{low}} = 0$ J.

![Fig. 6: Motion and force data for the wiping task experiment](image)

In the first phase of the experiment the desired polishing motion was demonstrated to the robot according to [9], encoded with periodic DMPs, and executed, see Fig. 8a. For better understanding the experimental results are divided into sections (Sec.) shown in Figs. 6, 7, 8, and in the accompanying video. Section 1 of the experiment shows the initial, stable, periodic polishing motion. $P_{\text{acv}}$ remains 0, the system passivity is preserved and no energy is drained from the tank. In Section 2 one corner of the table is lifted, introducing a change to the environment. Passivity (28) is violated and the DMP is slowed down. This gives time to the admittance controller to adapt to the change. We can see from the bottom plot in Fig. 7 that the tank energy is partially drained and filled up again in the same period of the motion, preserving the overall tank energy.

Section 3 depicts a stable motion. The other corner of the table is lifted, introducing a different change to the environment, see Fig. 8c. Passivity is preserved. In the adaptation process some energy from the tank was injected into the system. Therefore, the overall tank energy has reduced.

Section 4 shows a strong passivity violation, where one corner of the table is rapidly lifted. The tank energy is immediately fully drained and the system stops, preventing damage from the robot and the environment, respectively.

### VII. CONCLUSION

Overall, the presented method gives a robot the ability to follow desired wrench profiles in rather unknown environments by provably stable trajectory adaptation. The method consists of following parts:

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### REFERENCES


(a) **Sec. 1:** Robot executes demonstrated wiping motion.

(b) **Sec. 2:** Environment has changed in comparison to demonstrated motion. The robot has to adapt to the change.

(c) **Sec. 3:** Robot adapts to the change, tank energy and passivity ensured.

(d) **Sec. 4:** Robot stops, passivity is violated and tank energy is drained.

![Humanoid robot polishing a previously unknown environment.](image)

Fig. 8: Humanoid robot polishing a previously unknown environment.


